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How do bookmakers (or FdJ[1]) ALWAYS manage to win ?

[1] FdJ stands for **F**rançaise **d**es **J**eux, which is the only bookmaker authorised in France. This company runs a game (*"Cote & Match"*) which pretty looks like bookmakers games

1 Introduction

We suppose of course that the question relates to soccer play between two teams, and that 3 outcomes are possible : home win, draw or away win. These results are usually referred to as 1,N,2 (french notation which stands for 1,X,2 international notation).

In the lines to come, we will suppose that we address ONE special match. This assumption is necessary to make the explanations more simple (though still tough), but the reasoning extrapolates easily to any match. We will thus derive the analysis that bookmakers make (or should make in our sense) to always win against punters.

To answer the question, we need various information.

First, the bookmaker must have his own idea of results probabilities : punting is after all a contest between the bookmaker (who wants to rip off players) and the players who do not agree with that purpose, consider themselves smarter than the bookmaker (or try to be), and of course want to get their money back at least and even more if possible.

Second, it is necessary to know the bets repartition among 1,X,2 possibilities. Not only the number of them (this is of course obvious) but also the amount of money repartition.

For example, 50% of players may put small money on a home win, whereas 10% put big money on a different result (X or 2). Finally, this last one will gather the most money, and that's what effectively counts for the bookmaker when it comes to the evaluation of his earnings chances.

This being said, we will now address the core question which is bookmaker trick to fool players. It will require some mathematical developments (we don't know how to make it simpler). So the webmaster would want to warn readers that from now on the text may be painful for some of them. Most courageous of you can keep on reading (make sure to have one aspirin tube at hand)

2 Notations & variables

		Known		
Notations	meaning	By players	by bookmakers	by FdJ
N	Total number of punters	no	yes	no
n_i	Number of bets on result i (i=1,X ou 2)	no	yes	no
m_i	Average euro bet on result i	no	yes	no
x_i	Bookmaker's odds	yes	yes	yes
p_i	Probability of result i estimated by the bookmaker	no	yes	yes
q_i	Probability of result i estimated by the punters	difficult	yes	no
M	Total amount of euros bet on the match	no	yes	no
m	Average bet per punter over all results (1,X,2)	no	yes	no
α	The bookmaker's tax on bets (his margin !)	no	yes	yes

Recall : the bookmaker odds x_i are the numbers used to multiply the punter's bet to calculate punter's earnings if he predicted the good result.

Relations between variables :

variable	Is equal to
N	$\sum_{i=1}^{3} n_i$
q_i	$\frac{n_i}{N}$

variable	Is equal to
M	$\sum_{i=1}^{3} n_i m_i$
m	$\frac{1}{N}\sum_{i=1}^{3} n_i m_i$

comments :

- To decide if a variable is known or not in preceding table, we looked for each kind of player (bookmaker, punter, FdJ) if they have a mean, DURING BETTING PERIOD OF TIME, to know this variable.
- It happens that both player and FdJ who make their bets or fix their odds before the betting period are somewhat even
- The punter even has a slight advantage over FdJ : if he is smart, he will wait for the last minutes of betting period to make his bet in order to have as much information as possible.
- Bookmakers on the contrary can permanently adjust their odds until the betting period ending. So they have a lot of very informative data (especially the amounts of bets per each result) which gives them a huge advantage over punters
- FdJ however has access to all data after betting period and can thus use this experience for next betting days. The game is thus much unfair than it could seem at first sight. FdJ has also the possibility to cancel the bet on any match if it appears that its risks are too high (same for bookmakers, but of course denied to punters).

According to this first analysis on access to betting data, bookmakers have already many means to bias the game in their favor. But that's not the only mean they have available. They have above all the opportunity to decide the odds levels, and this is a tremendous weapon to make the balance shift their way. That's what we try to demonstrate in next paragraphs.

3 Bookmaker's benefit expected value

This value corresponds to the average benefit the bookmaker could make if the match considered was played many times.

In fact, the match is only played once, but as there are many matches, the computation of the "expected value" is nevertheless relevant.

This "expected value" is by convention written $E(X)$, where X is the probabilistic variable we look for, namely the bookmaker's benefit here. For instance, the expected value of the result of a roll of a 6 faces dice is 3.5; this doesn't mean you'll get 3.5 when you roll the dice (of course, or you have really weird dice) but that you'll get this average result if you roll the dice many times.

First, let's look at bookmaker's earnings for each game result :

Match result	Bookmaker's earning (eventually negative, i.e. = loss)
1	$n_1m_1(1-x_1)+n_2m_2+n_3m_3$
X	$n_1m_1+n_2m_2(1-x_2)+n_3m_3$
2	$n_1m_1+n_2m_2+n_3m_3(1-x_3)$

As the bookmaker's prediction for 1,X,2 results are p_1, p_2, p_3, his benefit expected value is finally :

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$$
\begin{aligned}
E(X) = {} & p_1.(n_1 m_1 (1-x_1) + n_2 m_2 + n_3 m_3) \\
& + p_2.(n_1 m_1 + n_2 m_2 (1-x_2) + n_3 m_3) \\
& + p_3.(n_1 m_1 + n_2 m_2 + n_3 m_3 (1-x_3))
\end{aligned}
$$

This can be rewritten using N, total number of bets on the match :

$$E(X) = N\left[p_1.\left(\frac{n_1}{N} m_1 (1-x_1) + \frac{n_2}{N} m_2 + \frac{n_3}{N} m_3\right) + p_2.\left(\frac{n_1}{N} m_1 + \frac{n_2}{N} m_2 (1-x_2) + \frac{n_3}{N} m_3\right) + p_3.\left(\frac{n_1}{N} m_1 + \frac{n_2}{N} m_2 + \frac{n_3}{N} m_3 (1-x_3)\right)\right]$$

$$E(X) = N\left[p_1.(q_1 m_1 (1-x_1) + q_2 m_2 + q_3 m_3) + p_2.(q_1 m_1 + q_2 m_2 (1-x_2) + q_3 m_3) + p_3.(q_1 m_1 + q_2 m_2 + q_3 m_3 (1-x_3))\right]$$

$$E(X) = N \left[\sum_{i=1}^{3} p_i \sum_{i=1}^{3} q_i m_i - \sum_{i=1}^{3} p_i q_i m_i x_i \right]$$

and finally, as $\sum_{i=1}^{3} p_i = 1$

$$\boxed{E(X) = N \left[\sum_{i=1}^{3} q_i m_i - \sum_{i=1}^{3} p_i q_i m_i x_i \right]} \qquad [1]$$

This is the most general formula for bookmaker's benefit expected value, as it does not include approximations or hypothesis of any kind

4 Bookmaker's strategies

In order to simplify the demonstration, we will assume that average amounts of euros bet on each result are the same. The bookmaker can have the actual data, is not obliged to go through this approximation, and can update the following reasoning as often as he wants.

We thus have : $m_1 = m_2 = m_3 = m$

The relation [1] becomes then a little simpler :

$$\boxed{E(X) = Nm \left[\sum_{i=1}^{3} q_i - \sum_{i=1}^{3} p_i q_i x_i \right] = Nm \left[1 - \sum_{i=1}^{3} p_i q_i x_i \right]} \qquad [1']$$

(because, as before, $\sum_{i=1}^{3} q_i = 1$)

The bookmaker, with this relation, can now develop his strategies.

4.1 *The honest bookmaker*

Of course, this variety of bookmaker does not exist, but his fictitious existence will help us explain further how "real life bookmakers" always manage to win.

So for this good ol' virtual philanthropist, nothing matters but making the game between him and punters fair. To reach this goal, he needs : $E(X) = 0$.

There's only one way to obtain this result, which is to choose the odds smartly. When looking to relation [1'], our naive bookmaker can observe that he has two obvious choices : $x_i = \frac{1}{q_i}$ or $x_i = \frac{1}{p_i}$.

In the first case, the relation [1'] becomes in fact :

$$E(X) = Nm\left[1 - \sum_{i=1}^{3} p_i q_i \frac{1}{q_i}\right] = Nm\left[1 - \sum_{i=1}^{3} p_i\right] = Nm\left[1 - 1\right] = 0$$

which is the objective that our bookmaker has fixed to himself..

In the second case, the result is the same because p_i et q_i act symmetrically in relation [1']. But the first solution is much more interesting for the bookmaker because he doesn't even have to make his own bets. Effectively, whatever the bookmaker's p_i values, even if they are very badly estimated, when choosing $x_i = \frac{1}{q_i}$ the final earning will be the same (win or loss equal to zero).

4.2 *"real life" bookmaker*

This bookmaker, that everyone knows, has two special characteristics that make him differ from the previous specimen :

- he must fix his odds BEFORE knowing the punters bets (so he doesn't know the q_i and must estimate them)
- he strongly whishes to have a positive benefit "expected value"

To solve his first problem, he as not much alternatives : all he can do is to suppose the punters to be as smart (or clueless) as he is, and assume that at the end of betting period he will have

$q_i = p_i$ (nevertheless, he will have the opportunity to scan the evolution of [1'] until the end, and to update the x_i so as to have *E(X)* remain positive).

With this assumption, his benefit "expected value" [1'] becomes :

$$E(X) = Nm\left[1 - \sum_{i=1}^{3} p_i p_i x_i\right] = Nm\left[1 - \sum_{i=1}^{3} p_i^2 x_i\right] \quad [1'']$$

and if he was the "honest" bookmaker, he would set $x_i = \dfrac{1}{p_i}$.

But he will not, because two things bother him much :

- First, he has no guarantee that punters will have the same predictions as him. This gives birth to a very embarrassing uncertainty on q_i, because it can make the relation [1"] not relevant, and relation [1'] (which is to be accounted for then) can be negative. Hence unpleasant loss in view !
- Second he is not pretty sure of his predictions p_i either, and would like to decrease this risk

So he will "work" his odds so as to guarantee a positive earning. Which means he will modify the x_i in order to get a margin on *E(X)*.

From now on, the true bookmakers methods can only be guesses. They can, for example, modify the three odds on (1,X,2) in the same way and calculate :

$$\boxed{x_i = \frac{1}{p_i}(1-\alpha)} \quad [2]$$

where α is chosen as a function of the margin the bookmakers wants for himself.

He will have the opportunity during the betting period to verify that the p_i and the q_i do not deviate too much from each other.

If they do, he will be able to "re-compute" the odds according to

$$\boxed{x_i = \frac{1}{q_i}(1-\alpha)} \quad [2']$$

in order to lower his risks (this is the case, because the q_i are known exactly and precisely). This will effectively guarantee him a positive benefit with minimum risk.

4.2.1 Benefit "expected value" under hypothesis [2] :

$$E(X) = Nm\left[1-\sum_{i=1}^{3} p_i^2 x_i\right] = Nm\left[1-\sum_{i=1}^{3} p_i^2 \frac{1}{p_i}(1-\alpha)\right] = Nm\left[1-(1-\alpha)\sum_{i=1}^{3}\frac{p_i^2}{p_i}\right]$$

$$E(X) = Nm\left[1-(1-\alpha)\sum_{i=1}^{3} p_i\right] = Nm\left[1-1+\alpha\right]$$

$$\boxed{E(X) = Nm\alpha}$$

where α is the bookmaker's tax on punters' bets [3]

4.2.2 Benefit "expected value" under hypothesis [2']

The bookmaker can greatly reduce his risks, as the q_i .are perfectly known.

His benefit "expected value" can then be calculated with relation [1'] which does not make any assumption on punters' bets and is thus much more precise. This expected value then writes :

$$E(X) = Nm\left[1-\sum_{i=1}^{3} p_i q_i x_i\right] = Nm\left[1-\sum_{i=1}^{3} p_i q_i \frac{1}{q_i}(1-\alpha)\right]$$

which gives the same expression [3] as for hypothesis [2], BUT without any hypothesis on q_i. This considerably reduces the bookmaker's risk..

In fact, the bookmaker can refine even more his strategy by using relation [1] which DOES NOT IMPLY ANY HYPOTHESIS AT ALL. He can thus compute his margins α_i with no error and propose very attractive odds.

4.3 La FdJ

This bookmaker never update his odds, and is obliged to play according to model [2]. His risks are higher, which explains for a part why the odds are less attractive than other online bookmakers.

5 How can we estimate the bookmaker's benefit ?

5.1 Margin estimation

Let's suppose that the bookmaker uses relation [2]. As we know his odds x_i, we can easily infer the "bet tax percentage" (and thus access to his "benefit expected value").

To do so, we must again assume a uniform average bet m over all results (1,X,2). From [2] we deduce

$$p_i = \frac{1-\alpha}{x_i}$$

and from $\sum_{i=1}^{3} p_i = 1$, we the get the formula to calculate α (unknown) from the x_i (known) :

$$\boxed{\alpha = 1 - \frac{1}{\sum_{i=1}^{3} \frac{1}{x_i}}} \qquad [4]$$

5.2 Examples (extracted from actual odds)

Odds France Ligue 1 , January 22nd to 23rd 2005.

FdJ :

teams		1	X	2
Metz	Marseille	2.7	2.65	2
St Etienne	PSG	1.95	2.6	2.85
Bastia	Nice	1.95	2.6	2.85
Caen	Auxerre	2.7	2.65	2
Istres	Strasbourg	1.95	2.6	2.85
Monaco	Lens	1.35	3.2	5.1
Rennes	Ajaccio	1.5	2.95	4.1
Sochaux	Bordeaux	1.85	2.65	3.05
Toulouse	Nantes	1.6	2.8	3.75
Lille	Lyon	2.4	2.55	2.3

Bookmaker :

teams		1	X	2
Metz	Marseille	2.7	2.9	2.55
St Etienne	PSG	2.55	2.9	2.7
Bastia	Nice	2.45	2.8	2.9
Caen	Auxerre	2.8	2.8	2.5
Istres	Strasbourg	2.4	2.9	2.9
Monaco	Lens	1.51	3.4	6.5
Rennes	Ajaccio	1.67	3	5.6
Sochaux	Bordeaux	2.25	2.8	3.25
Toulouse	Nantes	1.83	3	4.35
Lille	Lyon	2.65	2.8	2.65

We use relation [4] to get each bookmaker's benefit estimations, which gives match per match the two following tables:

FdJ :

Metz	Marseille	19.9%
St Etienne	PSG	19.9%
Bastia	Nice	19.9%
Caen	Auxerre	19.9%
Istres	Strasbourg	19.9%
Monaco	Lens	20.0%
Rennes	Ajaccio	20.0%
Sochaux	Bordeaux	19.7%
Toulouse	Nantes	19.9%
Lille	Lyon	19.6%

Bookmaker

Metz	Marseille	9.7%
St Etienne	PSG	9.7%
Bastia	Nice	9.9%
Caen	Auxerre	10.3%
Istres	Strasbourg	9.6%
Monaco	Lens	9.9%
Rennes	Ajaccio	10.0%
Sochaux	Bordeaux	9.9%
Toulouse	Nantes	9.9%
Lille	Lyon	10.1%

We have with this example a guess :

1. of the margin that online bookmakers make on punters bets (>10%, which is enormous speaking of revenue on money you don't own)
2. of the "uncertainty bonus" that FdJ grants to herself (+10% wrt bookmaker, which means a comfortable 20% !!!)